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Dynamics of two forced quantum oscillators with parametric down-conversion interaction solved by virtue of the entangled state representation*

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Abstract
By virtue of the entangled state representation $|\xi\rangle\rangle$, we solve the dynamics of a generalized parametric amplifier whose Hamiltonian is composed of two forced quantum oscillators plus a parametric down-conversion interaction in the resonant case. The solutions and state vectors of the Schrödinger equation are derived, of which the simplest solution is a squeezed coherent state. The method of characteristics is employed.

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1. Introduction

Since the publication of the paper of Einstein, Podolsky and Rosen (EPR) in 1935 [1], arguing the incompleteness of quantum mechanics, the conception of entanglement has become more and more fascinating and important as it plays a central role in quantum communication and quantum computation [2–5]. The two-mode squeezed state, generated from a parametric down-conversion process (in a non-degenerate parametric amplifier) with the Hamiltonian being [6]

$$\mathcal{H} = \omega_0\left(a_1^\dagger a_1 + a_2^\dagger a_2\right) + g\left(a_1^\dagger a_2^\dagger \exp(-\mathrm{i}2\omega_0 t) + a_1 a_2 \exp(\mathrm{i}2\omega_0 t)\right) \tag{1.1}$$

has its idler-mode photon and signal-mode photon entangled with each other in the frequency domain, i.e. the correlation between idler mode and signal mode gives rise to two-mode squeezing. Some new relationships between squeezing and entangled state transformation are further revealed in [7]. On the other hand, it has been shown that the forced quantum oscillator subjected to a transient 'classical' driving force can generate the coherent states [8],

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then a question naturally arises: can we combine the mechanism of the parametric down-conversion process and that of the forced quantum oscillator to directly produce a squeezed coherent state? To put it another way, when two forced quantum oscillators are coupled by some interactions which cause two-mode squeezing, what is the dynamic evolution? In this work we shall study how the dynamics of a time-dependent generalized parametric amplifier can be solved by virtue of the entangled state representation; the latter is constructed in [9, 10]. By the generalized parametric amplifier, we mean that its Hamiltonian describes a two-mode forced quantum oscillator with the parametric down-conversion interaction in the resonant case. In section 2 we briefly review the entangled state representation $|\xi\rangle\rangle$. In section 3 the characteristic equation deduced from the Schrödinger equation of the generalized parametric amplifier is solved, and its physical implementation is briefly discussed. The elegance and the efficiency working in the $|\xi\rangle\rangle$ representation lies in that the Schrödinger equation is projected as a first-order partial differential equation which can be solved with the method of characteristics. The physical interpretation of the solution is discussed in section 4.

2. Brief review of the entangled state representation

Similar to EPR's original idea that two particles' relative position operator commutes with their total momentum operator, two particles' coordinate operator sum Q_1+Q_2 also commutes with their relative momentum operator, i.e. $[Q_1+Q_2, P_1-P_2]=0$, and we can set up their common un-normalized eigenvector of continuous variable in the two-mode Fock space [7], i.e.

$$|\xi\rangle\rangle = \exp\left[\xi a_1^\dagger + \xi^* a_2^\dagger - a_1^\dagger a_2^\dagger\right]|00\rangle \qquad \xi = \xi_1 + \mathrm{i}\xi_2 \tag{2.1}$$

where the $a_1^\dagger$-mode ($a_2^\dagger$-mode) is the creation operator in the Fock space; ξ is a complex number whose real and imaginary parts are indeed the eigenvalues of Q_1+Q_2 and P_1-P_2, respectively,

$$(Q_1+Q_2)|\xi\rangle\rangle = \sqrt{2}\xi_1|\xi\rangle\rangle \qquad (P_1-P_2)|\xi\rangle\rangle = \sqrt{2}\xi_2|\xi\rangle\rangle. \tag{2.2}$$

By using $|00\rangle\langle 00| =: \exp\left(-a_1^\dagger a_1 - a_2^\dagger a_2\right):$, and the technique of integral within an order product (IWOP) of operators [11] we can prove the completeness relation of $|\xi\rangle\rangle$,

$$\int \frac{\mathrm{d}^2\xi}{\pi}\,\mathrm{e}^{-|\xi|^2}|\xi\rangle\rangle\langle\langle\xi| = \int \frac{\mathrm{d}^2\xi}{\pi} : \exp\left(-|\xi|^2 + \xi a_1^\dagger + \xi^* a_2^\dagger - a_1^\dagger a_2^\dagger \right.$$
$$\left. + \xi^* a_1 + \xi a_2 - a_1 a_2 - a_1^\dagger a_1 - a_2^\dagger a_2\right) := 1. \tag{2.3}$$

Thus $|\xi\rangle\rangle$ is qualified to be a quantum mechanical representation, and the following properties can be easily obtained from (2.1),

$$\begin{aligned} \langle\langle\xi|a_1^\dagger &= \left(\xi^* - \frac{\partial}{\partial\xi}\right)\langle\langle\xi| \qquad & \langle\langle\xi|a_1 &= \frac{\partial}{\partial\xi^*}\langle\langle\xi| \\ \langle\langle\xi|a_2^\dagger &= \left(\xi - \frac{\partial}{\partial\xi^*}\right)\langle\langle\xi| \qquad & \langle\langle\xi|a_2 &= \frac{\partial}{\partial\xi}\langle\langle\xi|. \end{aligned} \tag{2.4}$$

In the next section, we show that the dynamics of the generalized parametric amplifier can be solved in the entangled state representation $\langle\langle\xi|$.

3. Solution of the Schrödinger equation for the generalized parametric amplifier

The Hamiltonian of the generalized parametric amplifier is actually a two-mode forced quantum oscillator with the parametric down-conversion interaction, i.e.

$$H = H_1 + H_2 \tag{3.1}$$

$$\begin{aligned} H_1 &= \omega'\left(a_1^\dagger a_1 + a_2^\dagger a_2 + 1\right) + \sqrt{2}\lambda X_1 \cos\omega_1 t + \sqrt{2}\sigma X_2 \cos\omega_2 t \\ X_i &= \left(a_i^\dagger + a_i\right)\big/\sqrt{2} \quad i = 1, 2 \end{aligned} \tag{3.2}$$

$$H_2 = g\left(a_1^\dagger a_2^\dagger \exp(-\mathrm{i}2\omega_0 t) + a_1 a_2 \exp(\mathrm{i}2\omega_0 t)\right) \qquad g = \omega' - \omega_0 \tag{3.3}$$

where H_1 describes a two-mode forced quantum oscillator, $\sqrt{2}\lambda X_1 \cos\omega_1 t$ and $\sqrt{2}\sigma X_2 \cos\omega_2 t$ are the classical forces; H_2 describes the classical pump mode at frequency $2\omega_0$ interacting in a nonlinear optical medium with two modes at the same frequency ω', the coupling constant g is proportional to the second-order susceptibility of the medium and to the amplitude of the pump. It is possible to adjust the pump-mode frequency and/or the ω'-mode frequency such that $\omega' = g + \omega_0$, this is named in resonant case. Our aim is to demonstrate that, although (3.1)–(3.3) is more complicated than (1.1), such a time-dependent Hamiltonian can be exactly solved in terms of the entangled state representation. For this purpose, we rewrite (3.1)–(3.3) as

$$\begin{aligned} H &= H_0 + H' \qquad H_0 = \omega_0\left(a_1^\dagger a_1 + a_2^\dagger a_2 + 1\right) \\ H' &= \lambda\left(a_1^\dagger + a_1\right)\cos\omega_1 t + \sigma\left(a_2^\dagger + a_2\right)\cos\omega_2 t \\ &\qquad + g\left(a_1^\dagger \exp(-\mathrm{i}\omega_0 t) + a_2 \exp(\mathrm{i}\omega_0 t)\right)\left(a_1 \exp(\mathrm{i}\omega_0 t) + a_2^\dagger \exp(-\mathrm{i}\omega_0 t)\right). \end{aligned} \tag{3.4}$$

Turning to the interaction picture, we have the Schrödinger equation

$$\begin{aligned} \mathrm{i}\frac{\partial}{\partial t}|\Psi(t)\rangle_I &= \exp(\mathrm{i}H_0 t)H' \exp(-\mathrm{i}H_0 t)|\Psi(t)\rangle_I \\ &= \left[\lambda\left(a_1^\dagger \exp(\mathrm{i}\omega_0 t) + a_1 \exp(-\mathrm{i}\omega_0 t)\right)\cos\omega_1 t + \sigma\left(a_2^\dagger \exp(\mathrm{i}\omega_0 t)\right.\right. \\ &\qquad \left.\left. + a_2 \exp(-\mathrm{i}\omega_0 t)\right)\cos\omega_2 t + g\left(a_1^\dagger + a_2\right)\left(a_1 + a_2^\dagger\right)\right]|\Psi(t)\rangle_I. \end{aligned} \tag{3.5}$$

Projecting (3.5) on the entangled state basis $\langle\langle\xi|$ and introducing

$$\Psi^I(\xi, \xi^*, t) \equiv \langle\langle\xi|\Psi(t)\rangle_I \tag{3.6}$$

as well as using (2.4) we have

$$\begin{aligned} &(-\lambda \exp(\mathrm{i}\omega_0 t)\cos\omega_1 t + \sigma \exp(-\mathrm{i}\omega_0 t)\cos\omega_2 t)\frac{\partial\Psi^I}{\partial\xi} \\ &\qquad + (\lambda \exp(-\mathrm{i}\omega_0 t)\cos\omega_1 t - \sigma \exp(\mathrm{i}\omega_0 t)\cos\omega_2 t)\frac{\partial\Psi^I}{\partial\xi^*} - \mathrm{i}\frac{\partial\Psi^I}{\partial t} \\ &\quad = -\left[(\lambda\xi^* \cos\omega_1 t + \sigma\xi \cos\omega_2 t)\exp(\mathrm{i}\omega_0 t) + g\,|\xi|^2\right]\Psi^I \end{aligned} \tag{3.7}$$

which involves an unknown function Ψ^I and three independent variables ξ, ξ^* and t. We see that though the bilinear terms $\left(a_1^\dagger + a_2\right)\left(a_1 + a_2^\dagger\right)$ exist in (3.5), in the $\langle\langle\xi|$ representation (3.7) is just a first-order partial differential equation, so it can be solved with the aid of the method of characteristics[4] [12].

The characteristic equations for (3.7) are

$$\frac{\mathrm{d}\xi}{R(t)} = \frac{\mathrm{d}t}{-\mathrm{i}} \qquad R(t) \equiv -\lambda\, \exp(\mathrm{i}\omega_0 t)\cos\omega_1 t + \sigma \exp(-\mathrm{i}\omega_0 t)\cos\omega_2 t \tag{3.8}$$

$$\frac{\mathrm{d}\xi^*}{R^*(t)} = \frac{\mathrm{d}t}{\mathrm{i}} \tag{3.9}$$

[4] There is a concise introduction about the method of characteristics in [12].

$$\frac{\mathrm{d}\Psi^I}{S(\xi,\xi^*,t,\Psi^I)} = \frac{\mathrm{d}t}{-\mathrm{i}}$$
$$S(\xi,\xi^*,t,\Psi^I) \equiv -[(\lambda\xi^*\cos\omega_1 t + \sigma\xi\cos\omega_2 t)\exp(\mathrm{i}\omega_0 t) + g\,|\xi|^2]\Psi^I. \tag{3.10}$$

Equations (3.9) and (3.8) are conjugated to each other, so we only need to consider the solution of (3.8). It is easily seen that the solution to (3.8) is

$$\xi \equiv A - \alpha(t) \tag{3.11}$$

where A is an integration constant, and

$$\alpha(t) \equiv \alpha_1(t) + \alpha_2^*(t)$$
$$\alpha_1(t) \equiv \left[\frac{\exp(\mathrm{i}\omega_1 t)}{\omega_0+\omega_1} + \frac{\exp(-\mathrm{i}\omega_1 t)}{\omega_0-\omega_1}\right]\frac{\lambda\exp(\mathrm{i}\omega_0 t)}{2} \tag{3.12}$$
$$\alpha_2(t) \equiv \left[\frac{\exp(\mathrm{i}\omega_2 t)}{\omega_0+\omega_2} + \frac{\exp(-\mathrm{i}\omega_2 t)}{\omega_0-\omega_2}\right]\frac{\sigma\exp(\mathrm{i}\omega_0 t)}{2}.$$

Substituting (3.11) into (3.10) and taking into account the following equations

$$\lambda\exp(\mathrm{i}\omega_0 t)\cos\omega_1 t = -\mathrm{i}\frac{\mathrm{d}\alpha_1}{\mathrm{d}t} \qquad \sigma\exp(\mathrm{i}\omega_0 t)\cos\omega_2 t = -\mathrm{i}\frac{\mathrm{d}\alpha_2}{\mathrm{d}t} \tag{3.13}$$

which are deduced from (3.12), we have

$$\frac{\mathrm{d}\ln\Psi^I}{\mathrm{d}t} = -\frac{\mathrm{d}\alpha_1}{\mathrm{d}t}(A^*-\alpha^*) - \frac{\mathrm{d}\alpha_2}{\mathrm{d}t}(A-\alpha) - \mathrm{i}g(A-\alpha)(A^*-\alpha^*). \tag{3.14}$$

Integrating (3.14) yields

$$\ln\Psi^I - \ln C = -\mathrm{i}gtAA^* + A\left[\mathrm{i}g\int\alpha^*\,\mathrm{d}t - \alpha_2\right] + A^*\left[\mathrm{i}g\int\alpha\,\mathrm{d}t - \alpha_1\right]$$
$$- \mathrm{i}g\int\alpha\alpha^*\,\mathrm{d}t + \alpha_1\alpha_2 + \int\left(\alpha_1^*\frac{\mathrm{d}\alpha_1}{\mathrm{d}t} + \alpha_2^*\frac{\mathrm{d}\alpha_2}{\mathrm{d}t}\right)\mathrm{d}t. \tag{3.15}$$

Here, we have absorbed all the integration constants arising from the indefinite integrations $\int\alpha\,\mathrm{d}t$, $\int\alpha^*\,\mathrm{d}t$, $\int\alpha\alpha^*\,\mathrm{d}t$ and $\int\left(\alpha_1^*\frac{\mathrm{d}\alpha_1}{\mathrm{d}t} + \alpha_2^*\frac{\mathrm{d}\alpha_2}{\mathrm{d}t}\right)\mathrm{d}t$ into C. Integrating the right-hand side of (3.15) and replacing the integration constants A and A^* by ξ and ξ^* according to (3.11), we obtain

$$\ln\Psi^I - \ln C = \beta\xi\xi^* + \gamma\xi + \delta\xi^* + \epsilon \tag{3.16}$$

where $\beta, \gamma, \delta, \epsilon$ are defined as

$$\beta(t) \equiv -\mathrm{i}gt$$
$$\gamma(t) \equiv -\alpha_2 - \mathrm{i}gt\alpha^* + \mathrm{i}g\int\alpha^*\,\mathrm{d}t$$
$$\delta(t) \equiv -\alpha_1 - \mathrm{i}gt\alpha + \mathrm{i}g\int\alpha\,\mathrm{d}t \tag{3.17}$$
$$\epsilon(t) \equiv -\alpha_1\alpha_2 - \int\left(\alpha_1\frac{\mathrm{d}\alpha_1^*}{\mathrm{d}t} + \alpha_2\frac{\mathrm{d}\alpha_2^*}{\mathrm{d}t}\right)\mathrm{d}t$$
$$+\mathrm{i}g\left[-t\alpha\alpha^* + \alpha\int\alpha^*\,\mathrm{d}t + \alpha^*\int\alpha\,\mathrm{d}t - \int\alpha\alpha^*\,\mathrm{d}t\right].$$

From (3.16), we derive

$$C = \Psi^I\exp(-\beta\xi\xi^* - \gamma\xi - \delta\xi^* - \epsilon) \equiv v(\Psi^I,\xi,\xi^*,t). \tag{3.18}$$

$v(\Psi^I, \xi, \xi^*, t)$ and $u(\xi, t) \equiv \xi + \alpha = A$ are two independent integral constants of the characteristic equations. According to the method of characteristics, we need a specific function $F(u, v) = 0$ to match with (3.7). Without loss of generality, we assume that $F(u, v)$ has the form

$$F(u, v) = v - f(u) = 0 \tag{3.19}$$

where f is a specific function which can be determined as follows. Noting that when $\sigma = 0, \lambda = 0, g = 0$, the time-dependent parameters $\alpha_{1,}\alpha_2, \beta, \gamma, \delta, \epsilon$ all come to zero, and the system reduces to a two-mode simple harmonic oscillator whose eigenstate vector is

$$|n_1, n_2\rangle = \frac{a_1^{\dagger n_1} a_2^{\dagger n_2}}{\sqrt{n_1! n_2!}} |00\rangle \tag{3.20}$$

in this case

$$u \to \xi \qquad v \to \Psi^I \equiv \langle\langle \xi | \Psi(t) \rangle_I = \langle\langle \xi | n_1, n_2 \rangle. \tag{3.21}$$

Substituting (3.21) into (3.19) we know

$$f(\xi) = \langle\langle \xi | n_1, n_2 \rangle. \tag{3.22}$$

Thus the form of $f(u)$ is

$$f(u) = \langle\langle u | n_1, n_2 \rangle. \tag{3.23}$$

So, from (3.19), (3.18) and (3.23) we deduce

$$\Psi^I = \langle\langle \xi | \Psi(t) \rangle_I = \exp(\beta \xi \xi^* + \gamma \xi + \delta \xi^* + \epsilon) \langle\langle \xi + \alpha | n_1, n_2 \rangle \tag{3.24}$$

which is the exact solution of (3.7). From (3.1), (3.2), we see that the physical implementation of the calculation is based on a set-up of the usual non-degenerate parametric amplifier, described by $\omega' \left(a_1^\dagger a_1 + a_2^\dagger a_2 + 1\right) + g\left(a_1^\dagger a_2^\dagger \exp(-\mathrm{i}2\omega_0 t) + a_1 a_2 \exp(\mathrm{i}2\omega_0 t)\right)$, influenced by two external periodic electric fields $E_i \sim a_i + a_i^\dagger$, $i = 1, 2$. Because the coupling g is proportional to the second-order susceptibility of the nonlinear medium and to the amplitude of the pump, one can either choose the medium or adjust the pump intensity to maintain the resonant condition $g = \omega' - \omega_0$.

4. Physical interpretation

Using the completeness relation of $\langle\langle \xi |$ shown in (2.3) and we can derive the corresponding state vector for (3.24),

$$\begin{aligned}
|\Psi(t)\rangle_I &= \int \frac{\mathrm{d}^2 \xi}{\pi} \mathrm{e}^{-|\xi|^2} |\xi\rangle\rangle \langle\langle \xi | \Psi(t) \rangle_I \\
&= \int \frac{\mathrm{d}^2 \xi}{\pi} \mathrm{e}^{-|\xi|^2} |\xi\rangle\rangle \exp(\beta \xi \xi^* + \gamma \xi + \delta \xi^* + \epsilon) \langle\langle \xi + \alpha | \, n_1, n_2 \rangle \\
&= \exp\left[\beta\left(a_1 + a_2^\dagger\right)\left(a_1^\dagger + a_2\right) + \gamma\left(a_1 + a_2^\dagger\right) + \delta\left(a_1^\dagger + a_2\right) + \epsilon\right] \exp[a_1 \alpha^* + a_2 \alpha] |n_1, n_2\rangle \\
&= \exp[\epsilon + \gamma \delta] \exp\left[\beta\left(a_1 + a_2^\dagger\right)\left(a_1^\dagger + a_2\right)\right] \exp\left[\delta a_1^\dagger + \gamma a_2^\dagger\right] \\
&\quad \times \exp[(\gamma + \alpha^*) a_1 + (\delta + \alpha) a_2] |n_1, n_2\rangle.
\end{aligned} \tag{4.1}$$

In order to see its physical meaning more clearly, we further put $\exp\left[\beta\left(a_1 + a_2^\dagger\right)\left(a_1^\dagger + a_2\right)\right]$ in its normal ordering form by virtue of the technique of integral within an ordered product of operators [11],

$$
\begin{aligned}
\exp\left[\beta\left(a_1+a_2^\dagger\right)\left(a_1^\dagger+a_2\right)\right] &= \int \frac{d^2\xi}{\pi}\exp(-(1-\beta)\,|\xi|^2)|\xi\rangle\rangle\langle\langle\xi| \\
&= \int \frac{d^2\xi}{\pi} : \exp\left[-(1-\beta)\,|\xi|^2 \right. \\
&\quad \left. +\xi\left(a_1^\dagger+a_2\right)+\xi^*\left(a_1+a_2^\dagger\right)-\left(a_1^\dagger+a_2\right)\left(a_1+a_2^\dagger\right)\right] : \\
&= : \frac{1}{1-\beta}\exp\left[\frac{\beta\left(a_1^\dagger+a_2\right)\left(a_1+a_2^\dagger\right)}{1-\beta}\right] : .
\end{aligned} \tag{4.2}
$$

When $n_1=0$, $n_2=0$, (4.1) becomes

$$
\begin{aligned}
|\Psi(t)\rangle_I &= \exp[\epsilon+\gamma\delta] : \left(\frac{1}{1-\beta}\right)\exp\left[\frac{\beta\left(a_1^\dagger+a_2\right)\left(a_1+a_2^\dagger\right)}{1-\beta}\right] : |\delta,\gamma\rangle \\
&= \frac{1}{1-\beta}\exp[\epsilon+\gamma\delta]\exp\left[\frac{\beta\left(a_1^\dagger+\gamma\right)\left(\delta+a_2^\dagger\right)}{1-\beta}\right]|\delta,\gamma\rangle
\end{aligned} \tag{4.3}
$$

where $|\delta,\gamma\rangle \equiv \exp\left[\delta a_1^\dagger+\gamma a_2^\dagger\right]|0,0\rangle$ is the two-mode un-normalized coherent state. Equation (4.3) shows that due to the existence of $\beta a_1^\dagger a_2^\dagger/(1-\beta)$, $\beta(t)\equiv -\mathrm{i}gt$, the Schrödinger state vector is a squeezed coherent state. In particular, if the parameter g in H, which represents the parametric amplifier interaction, is zero, then $H_2=0$; from (3.17), we see that (4.1) reduces to the solution for the two forced quantum oscillators

$$
|\Psi(t)\rangle_I = |\phi(t)\rangle_1 \otimes |\phi(t)\rangle_2 \tag{4.4}
$$

where

$$
|\phi(t)\rangle_i = \exp\left[-\int \alpha_i \frac{d\alpha_i^*}{dt}\,dt+\frac{1}{2}\alpha_i\alpha_i^*\right]\exp\left[-\alpha_i a_i^\dagger+\alpha_i^* a_i\right]|n_i\rangle. \tag{4.5}
$$

Let $D(-\alpha_i)=\exp\left[-\alpha_i a_i^\dagger+\alpha_i^* a_i\right]$ be the displacement operator, then

$$
|\phi(t)\rangle_i = \exp\left[-\int \alpha_i \frac{d\alpha_i^*}{dt}\,dt+\frac{1}{2}\alpha_i\alpha_i^*\right]D(-\alpha_i)|n_i\rangle. \tag{4.6}
$$

$D(-\alpha_i)|n_i\rangle$ is a displaced Fock state, and can be further expressed as

$$
D(-\alpha_i)|n_i\rangle = \frac{1}{\sqrt{n_i!}}\left(a_i^\dagger+\alpha_i^*\right)^{n_i}|-\alpha_i\rangle. \tag{4.7}
$$

$|-\alpha_i\rangle$ is a normalized coherent state [13, 14], which coincides with the conclusion about forced quantum oscillators in [8].

In summary, we have shown that the dynamics of two forced quantum oscillators with parametric down-conversion interaction in the resonant case can be exactly solved by virtue of the entangled state representation. This dynamics is directly leading to production of squeezed coherent states.The convenience and efficiency of working in this representation lies in that the Schrödinger equation is reduced to a first-order partial differential equation which can be solved through the method of characteristics. Thus this work together with [15], where the entangled states are employed to deal with master equations, exhibits the merit of the entangled state representation used in the quantum optics theory.

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